# Package 'grpnet'

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Type Package

Title Group Elastic Net Regularized GLMs and GAMs

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<b>Description</b> Efficient algorithms for fitting generalized linear and additive models with group elastic net penalties as described in Helwig (2024) <doi:10.1080 10618600.2024.2362232="">. Implements group LASSO, group MCP, and group SCAD with an optional group ridge penalty. Computes the regularization path for linear regression (gaussian), logistic regression (binomial), multinomial logistic regression (multinomial), log-linear count regression (poisson and negative.binomial), and log-linear continuous regression (gamma and inverse gaussian). Supports default and formula methods for model specification, k-fold crossvalidation for tuning the regularization parameters, and nonparametric regression via tensor product reproducing kernel (smoothing spline) basis function expansion.</doi:10.1080>
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## **Description**

auto

Miles per gallon and other characteristics of vehicles from the 1970s-1980s. A version of this dataset was used as the 1983 American Statistical Association Exposition dataset.

## Usage

```
data("auto")
```

#### **Format**

A data frame with 392 observations on the following 9 variables.

```
mpg miles per gallon (numeric vector)
cylinders number of cylinders: 3,4,5,6,8 (ordered factor)
displacement engine displacement in cubic inches (numeric vector)
horsepower engine horsepower (integer vector)
weight vehicle weight in of lbs. (integer vector)
acceleration 0-60 mph time in sec. (numeric vector)
model.year ranging from 1970 to 1982 (integer vector)
origin region of origin: American, European, Japanese (factor vector)
```

#### **Details**

This is a modified version of the "Auto MPG Data Set" on the UCI Machine Learning Repository, which is a modified version of the "cars" dataset on StatLib.

Compared to the version of the dataset in UCI's MLR, this version of the dataset has removed (i) the 6 rows with missing horsepower scores, and (ii) the last column giving the name of each vehicle (car.name).

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## **Source**

The dataset was originally collected by Ernesto Ramos and David Donoho.

StatLib—Datasets Archive at Carnegie Mellon University http://lib.stat.cmu.edu/datasets/cars.data

Machine Learning Repository at University of California Irvine https://archive.ics.uci.edu/ml/datasets/Auto+MPG

## **Examples**

```
# load data
data(auto)

# display structure
str(auto)

# display header
head(auto)

# see 'cv.grpnet' for cross-validation examples
?cv.grpnet

# see 'grpnet' for fitting examples
?grpnet
```

coef

Extract Coefficients for cv.grpnet and grpnet Fits

## **Description**

Obtain coefficients from a cross-validated group elastic net regularized GLM (cv.grpnet) or a group elastic net regularized GLM (grpnet) object.

## Usage

```
## S3 method for class 'cv.grpnet'
coef(object,
    s = c("lambda.min", "lambda.1se"),
    ...)
## S3 method for class 'grpnet'
coef(object,
    s = NULL,
    ...)
```

## **Arguments**

object Object of class "cv.grpnet" or "grpnet"

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s Lambda value(s) at which predictions should be obtained. For "cv.grpnet" objects, default uses the lambda that minimizes the cross-validation loss function. For "grpnet" objects, default uses s = object\$lambda. Interpolation is used for s values that are not included in object\$lambda.

... Additional arguments (ignored)

#### **Details**

coef.cv.grpnet:

Returns the coefficients that are used by the predict.cv.grpnet function to form predictions from a fit cv.grpnet object.

coef.grpnet:

Returns the coefficients that are used by the predict.grpnet function to form predictions from a fit grpnet object.

#### Value

For multinomial response variables, returns a list of length length(object\$ylev), where the j-th element is a matrix of dimension c(ncoef, length(s)) giving the coefficients for object\$ylev[j].

For other response variables, returns a matrix of dimension c(ncoef, length(s)), where the i-th column gives the coefficients for s[i].

#### Note

The syntax of these functions closely mimics that of the coef.cv.glmnet and coef.glmnet functions in the **glmnet** package (Friedman, Hastie, & Tibshirani, 2010).

#### Author(s)

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## References

Friedman, J., Hastie, T., & Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. *Journal of Statistical Software*, *33*(1), 1-22. doi:10.18637/jss.v033.i01

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. *Journal of Computational and Graphical Statistics*. doi:10.1080/10618600.2024.2362232

## See Also

```
print.coef.grpnet for printing coef.grpnet objects
predict.cv.grpnet for predicting from cv.grpnet objects
predict.grpnet for predicting from grpnet objects
```

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## **Examples**

```
#####***######
                  grpnet #####***#####
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto)</pre>
# extract coefs for regularization path (output = 12 x 100 matrix)
coef(mod)
# extract coefs at 3 particular points (output = 12 x 3 matrix)
coef(mod, s = c(1.5, 1, 0.5))
######**
                 cv.grpnet #####***#####
# load data
data(auto)
# 5-fold cv (formula method, response = mpg)
mod <- cv.grpnet(mpg ~ ., data = auto, nfolds = 5, alpha = 1)</pre>
# extract coefs for "min" solution (output = 12 x 1 matrix)
coef(mod)
# extract coefs for "1se" solution (output = 12 x 1 matrix)
coef(mod, s = "lambda.1se")
# extract coefs at 3 particular points (output = 12 x 3 matrix)
coef(mod, s = c(1.5, 1, 0.5))
```

cv.compare

Compare Multiple cv.grpnet Solutions

## **Description**

Creates a plot (default) or returns a data frame (otherwise) that compares the cross-validation error for multiple cv.grpnet fits.

# Usage

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```
point.col = "red",
line.col = "gray",
lwd = 2,
bwd = 0.02,
labels = NULL,
xlim = NULL,
ylim = NULL,
xlab = NULL,
ylab = NULL,
...)
```

## **Arguments**

X	a single cv.grpnet object or a list of cv.grpnet objects.
S	the tuning parameter value at which to plot results (if x is a list).
plot	switch controlling whether a plot is produced (default) versus data frame.
at	x-axis coordinates for plotting the cv error for each solution.
nse	number of standard errors to use for error bars in plot.
point.col	color for point used to plot the average of the cv error.
line.col	color for lines used to plot the standard error for the cv error.
lwd	width of lines used to plot the standard error for the cv error.
bwd	width of standard error bars in terms of proportion of range(x).
labels	labels for x-axis tick marks. Defaults to names(x).
xlim	axis limits for abscissa (x-axis)
ylim	axis limits for ordinate (y-axis)
xlab	axis label for abscissa (x-axis)
ylab	axis label for ordinate (y-axis)
	additional arguments passed to plotting functions.

## **Details**

Default behavior creates a plot that displays the mean cv error +/- 1 se for each of the requested solutions.

If the input x is a single cv.grpnet object, then the function plots the lambda.min and lambda.1se solutions.

If the input x is a list of cv.grpnet objects, then the function plots either the lambda.min or the lambda.1se solution (controlled by s argument) for all of the input models.

## Value

```
When plot = TRUE, there is no return value (it produces a plot)
```

When plot = FALSE, a data.frame is returned with the mean cv error (and se) for each solution

## Author(s)

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#### References

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. *Journal of Computational and Graphical Statistics*. doi:10.1080/10618600.2024.2362232

#### See Also

```
plot.cv.grpnet for plotting cv error path (for all lambdas)
plot.grpnet for plotting regularization path (for single lambda)
```

## **Examples**

```
# load data
data(auto)

# LASSO penalty
set.seed(1)
mod1 <- cv.grpnet(mpg ~ ., data = auto, nfolds = 5, alpha = 1)

# MCP penalty
set.seed(1)
mod2 <- cv.grpnet(mpg ~ ., data = auto, nfolds = 5, alpha = 1, penaly = "MCP")

# SCAD penalty
set.seed(1)
mod3 <- cv.grpnet(mpg ~ ., data = auto, nfolds = 5, alpha = 1, penaly = "SCAD")

# compare lambda.min and lambda.1se for mod1
cv.compare(mod1)

# compare lambda.1se for mod1, mod2, mod3
cv.compare(x = list(mod1, mod2, mod3), labels = c("LASSO", "MCP", "SCAD"))</pre>
```

cv.grpnet

Cross-Validation for grpnet

## **Description**

Implements k-fold cross-validation for grpnet to find the regularization parameters that minimize the prediction error (deviance, mean squared error, mean absolute error, or misclassification rate).

## Usage

```
cv.grpnet(x, ...)
## Default S3 method:
cv.grpnet(x,
          у,
          group,
          weights = NULL,
          offset = NULL,
          alpha = c(0.01, 0.25, 0.5, 0.75, 1),
          gamma = c(3, 4, 5),
          type.measure = NULL,
          nfolds = 10,
          foldid = NULL,
          same.lambda = FALSE,
          parallel = FALSE,
          cluster = NULL,
          verbose = interactive(),
          ...)
## S3 method for class 'formula'
cv.grpnet(formula,
          data,
          use.rk = TRUE,
          weights = NULL,
          offset = NULL,
          alpha = c(0.01, 0.25, 0.5, 0.75, 1),
          gamma = c(3, 4, 5),
          type.measure = NULL,
          nfolds = 10,
          foldid = NULL,
          same.lambda = FALSE,
          parallel = FALSE,
          cluster = NULL,
          verbose = interactive(),
          ...)
```

## **Arguments**

X	Model (design) matrix of dimension nobs by nitration $(n \times p)$ .
У	Response vector of length $n$ . Matrix inputs are allowed for binomial and multinomial families (see "Binomial and multinomial" section in grpnet).
group	Group label vector (factor, character, or integer) of length $p$ . Predictors with the same label are grouped together for regularization.
formula	Model formula: a symbolic description of the model to be fitted. Uses the same syntax as 1m and g1m.
data	Optional data frame containing the variables referenced in formula.

use.rk	If TRUE (default), the rk.model.matrix function is used to build the model matrix. Otherwise, the model.matrix function is used to build the model matrix. Additional arguments to the rk.model.matrix function can be passed via the argument.
weights	Optional vector of length $\boldsymbol{n}$ with non-negative weights to use for weighted (penalized) likelihood estimation. Defaults to a vector of ones.
offset	Optional vector of length $\boldsymbol{n}$ with an a priori known term to be included in the model's linear predictor. Defaults to a vector of zeros.
alpha	Scalar or vector specifying the elastic net tuning parameter $\alpha$ . If alpha is a vector (default), then (a) the same foldid is used to compute the cross-validation error for each $\alpha$ , and (b) the solution for the optimal $\alpha$ is returned.
gamma	Scalar or vector specifying the penalty hyperparameter $\gamma$ for MCP or SCAD. If gamma is a vector (default), then (a) the same foldid is used to compute the cross-validation error for each $\gamma$ , and (b) the solution for the optimal $\gamma$ is returned.
type.measure	Loss function for cross-validation. Options include: "deviance" for model deviance, "mse" for mean squared error, "mae" for mean absolute error, or "class" for classification error. Note that "class" is only available for binomial and multinomial families. The default is classification error (for binomial and multinomial) or deviance (others).
nfolds	Number of folds for cross-validation.
foldid	Optional vector of length $n$ giving the fold identification for each observation. Must be coercible into a factor. After coersion, the nfolds argument is defined as nfolds = nlevels(foldid).
same.lambda	Logical specfying if the same $\lambda$ sequence should be used for fitting the model to each fold's data. If FALSE (default), the $\lambda$ sequence is determined separately holding out each fold, and the $\lambda$ sequence from the full model is used to align the predictions. If TRUE, the $\lambda$ sequence from the full model is used to fit the model for each fold. The default often provides better (i.e., more stable) computational performance.
parallel	Logical specifying if sequential computing (default) or parallel computing should be used. If TRUE, the fitting for each fold is parallelized.
cluster	Optional cluster to use for parallel computing. If parallel = TRUE and cluster = NULL, then the cluster is defined cluster = makeCluster(2L), which uses two cores. Recommended usage: cluster = makeCluster(detectCores())
verbose	Logical indicating if the fitting progress should be printed. Defaults to TRUE in interactive sessions and FALSE otherwise.
	Optional additional arguments for $\ensuremath{grpnet}$ (e.g., standardize, penalty.factor, etc.)

# **Details**

This function calls the grpnet function nfolds+1 times: once on the full dataset to obtain the lambda sequence, and once holding out each fold's data to evaluate the prediction error. The syntax

of (the default S3 method for) this function closely mimics that of the cv.glmnet function in the **glmnet** package (Friedman, Hastie, & Tibshirani, 2010).

Let  $\mathbf{D}_u = \{\mathbf{y}_u, \mathbf{X}_u\}$  denote the u-th fold's data, let  $\mathbf{D}_{[u]} = \{\mathbf{y}_{[u]}, \mathbf{X}_{[u]}\}$  denote the full dataset excluding the u-th fold's data, and let  $\boldsymbol{\beta}_{\lambda[u]}$  denote the coefficient estimates obtained from fitting the model to  $\mathbf{D}_{[u]}$  using the regularization parameter  $\lambda$ .

The cross-validation error for the u-th fold is defined as

$$E_u(\lambda) = C(\boldsymbol{\beta}_{\lambda[u]}, \mathbf{D}_u)$$

where  $C(\cdot,\cdot)$  denotes the cross-validation loss function that is specified by type.measure. For example, the "mse" loss function is defined as

$$C(\boldsymbol{\beta}_{\lambda[u]}, \mathbf{D}_u) = \|\mathbf{y}_u - \mathbf{X}_u \boldsymbol{\beta}_{\lambda[u]}\|^2$$

where  $\|\cdot\|$  denotes the L2 norm.

The mean cross-validation error cvm is defined as

$$\bar{E}(\lambda) = \frac{1}{v} \sum_{u=1}^{v} E_u(\lambda)$$

where v is the total number of folds. The standard error cvsd is defined as

$$S(\lambda) = \sqrt{\frac{1}{v(v-1)} \sum_{u=1}^{v} (E_u(\lambda) - \bar{E}(\lambda))^2}$$

which is the classic definition of the standard error of the mean.

## Value

lambda	regularization parameter sequence for the full data
CVM	mean cross-validation error for each lambda
cvsd	estimated standard error of cvm
cvup	upper curve: cvm + cvsd
cvlo	lower curve: cvm - cvsd
nzero	number of non-zero groups for each lambda
grpnet.fit	fitted grpnet object for the full data
lambda.min	value of lambda that minimizes cvm
lambda.1se	largest lambda such that cvm is within one cvsd from the minimum (see Note)
index	two-element vector giving the indices of lambda.min and lambda.1se in the lambda vector, i.e., c(minid, se1id) as defined in the Note
type.measure	loss function for cross-validation (used for plot label)
call	matched call
tune	data frame containing the tuning results, i.e., min(cvm) for each combo of alpha and/or gamma

#### Note

```
lambda.1se is defined as follows:
minid <- which.min(cvm)
min1se <- cvm[minid] + cvsd[minid]
se1id <- which(cvm <= min1se)[1]
lambda.1se <- lambda[se1id]</pre>
```

#### Author(s)

Nathaniel E. Helwig <helwig@umn.edu>

#### References

Breheny, P., & Huang, J. (2015). Group descent algorithms for nonconvex penalized linear and logistic regression models with grouped predictors. *Statistics and Computing*, 25, 173-187. doi:10.1007/s1122201394242

Friedman, J., Hastie, T., & Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. *Journal of Statistical Software*, *33*(1), 1-22. doi:10.18637/jss.v033.i01

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. *Journal of Computational and Graphical Statistics*. doi:10.1080/10618600.2024.2362232

Yang, Y., & Zou, H. (2015). A fast unified algorithm for solving group-lasso penalize learning problems. *Statistics and Computing*, 25, 1129-1141. doi:10.1007/s1122201494985

## See Also

```
plot.cv.grpnet for plotting the cross-validation error curve
predict.cv.grpnet for predicting from cv.grpnet objects
grpnet for fitting group elastic net regularization paths
```

## **Examples**

```
######****##### family = "gaussian" #####***#####

# load data
data(auto)

# 5-fold cv (formula method, response = mpg)
set.seed(1)
mod <- cv.grpnet(mpg ~ ., data = auto, nfolds = 5, alpha = 1)

# print min and 1se solution info
mod

# plot cv error curve
plot(mod)</pre>
```

```
######**
                family = "binomial" #####***#####
# load data
data(auto)
# define response (1 = American, 0 = other)
y <- ifelse(auto$origin == "American", 1, 0)
# define predictors
x \leftarrow rk.model.matrix(~0 + ., data = auto[,1:7])
# define group
g <- attr(x, "assign")</pre>
# 10-fold cv (default method, response = y)
set.seed(1)
mod <- cv.grpnet(x, y, g, family = "binomial", alpha = 1)</pre>
# print min and 1se solution info
mod
# plot cv error curve
plot(mod)
#####***#### family = "poisson" #####***#####
# load data
data(auto)
# 10-fold cv (formula method, response = horsepower)
set.seed(1)
mod <- cv.grpnet(horsepower ~ ., data = auto, family = "poisson", alpha = 1)</pre>
# print min and 1se solution info
mod
# plot cv error curve
plot(mod)
#####***##### family = "negative.binomial" #####***#####
# load data
data(auto)
# 10-fold cv (formula method, response = horsepower)
set.seed(1)
mod <- cv.grpnet(horsepower ~ ., data = auto, family = "negative.binomial",</pre>
                 alpha = 1, theta = 100)
```

```
# print min and 1se solution info
mod
# plot cv error curve
plot(mod)
#####***#### family = "multinomial" #####***#####
# load data
data(auto)
# 10-fold cv (formula method, response = origin)
mod <- cv.grpnet(origin ~ ., data = auto, family = "multinomial", alpha = 1)</pre>
# print min and 1se solution info
mod
# plot cv error curve
plot(mod)
#####***##### family = "Gamma" #####***#####
# load data
data(auto)
# 10-fold cv (formula method, response = origin)
mod <- cv.grpnet(mpg ~ ., data = auto, family = "Gamma", alpha = 1)</pre>
# print min and 1se solution info
mod
# plot cv error curve
plot(mod)
#####***##### family = "inverse.gaussian" #####***#####
# load data
data(auto)
# 10-fold cv (formula method, response = origin)
set.seed(1)
mod <- cv.grpnet(mpg ~ ., data = auto, family = "inverse.gaussian", alpha = 1)</pre>
# print min and 1se solution info
```

```
mod
# plot cv error curve
plot(mod)
```

grpnet

Fit a Group Elastic Net Regularized GLM/GAM

## **Description**

Fits generalized linear/additive models with a group elastic net penalty. Predictor groups can be manually input (default S3 method) or inferred from the model (S3 "formula" method). The regularization path is computed at a data-generated (default) or user-provided sequence of lambda values.

## Usage

```
grpnet(x, ...)
## Default S3 method:
grpnet(x,
       у,
       group,
       family = c("gaussian", "binomial", "multinomial", "poisson",
                   "negative.binomial", "Gamma", "inverse.gaussian"),
       weights = NULL,
       offset = NULL,
       alpha = 1,
       nlambda = 100.
       lambda.min.ratio = ifelse(nobs < nvars, 0.05, 0.0001),</pre>
       lambda = NULL,
       penalty.factor = NULL,
       penalty = c("LASSO", "MCP", "SCAD"),
       gamma = 4,
       theta = 1,
       standardized = !orthogonalized,
       orthogonalized = TRUE,
       intercept = TRUE,
       thresh = 1e-04,
       maxit = 1e05,
       ...)
## S3 method for class 'formula'
grpnet(formula,
       data,
```

```
use.rk = TRUE,
family = c("gaussian", "binomial", "multinomial", "poisson",
           "negative.binomial", "Gamma", "inverse.gaussian"),
weights = NULL,
offset = NULL,
alpha = 1,
nlambda = 100,
lambda.min.ratio = ifelse(nobs < nvars, 0.05, 0.0001),
lambda = NULL.
penalty.factor = NULL,
penalty = c("LASSO", "MCP", "SCAD"),
gamma = 4,
theta = 1,
standardized = !orthogonalized,
orthogonalized = TRUE,
thresh = 1e-04,
maxit = 1e05,
...)
```

#### **Arguments**

x Model (design) matrix of dimension nobs by nvars  $(n \times p)$ .

y Response vector of length n. Matrix inputs are allowed for binomial and multi-

nomial families (see "Binomial and multinomial" section).

group Group label vector (factor, character, or integer) of length p. Predictors with the

same label are grouped together for regularization.

formula Model formula: a symbolic description of the model to be fitted. Uses the same

syntax as 1m and g1m.

data Optional data frame containing the variables referenced in formula.

use.rk If TRUE (default), the rk.model.matrix function is used to build the model matrix. Otherwise, the model.matrix function is used to build the model matrix.

Additional arguments to the rk.model.matrix function can be passed via the

... argument.

family Character specifying the assumed distribution for the response variable. Par-

tial matching is allowed. Options include "gaussian" (real-valued response), "binomial" (binary response), "multinomial" (multi-class response), "poisson" (count response), "negative.binomial" (count response), "Gamma" (positive

real-valued), or "inverse.gaussian" (positive real-valued).

weights Optional vector of length n with non-negative weights to use for weighted (pe-

nalized) likelihood estimation. Defaults to a vector of ones.

offset Optional vector of length n with an a priori known term to be included in the

model's linear predictor. Defaults to a vector of zeros.

alpha Regularization hyperparameter satisfying  $0 \le \alpha \le 1$  that gives the balance

between the group L1 (lasso) and group L2 (ridge) penalty. Setting  $\alpha=1$  uses a group lasso penalty, setting  $\alpha=0$  uses a group ridge penalty, and setting

 $0 < \alpha < 1$  uses a group elastic net group penalty.

nlambda

Number of  $\lambda$  values to use in the regularization path. Ignored if lambda is provided.

lambda.min.ratio

The proportion  $0 < \pi < 1$  that defines the minimum regularization parameter  $\lambda_{\min}$  as a fraction of the maximum regularization parameter  $\lambda_{\max}$  via the relationship  $\lambda_{\min} = \pi \lambda_{\max}$ . Ignored if lambda is provided. Note that  $\lambda_{\max}$  is defined such that all penalized effects are shrunk to zero.

lambda

Optional vector of user-supplied regularization parameter values.

penalty.factor

Default S3 method: vector of length K giving the non-negative penalty weight for each predictor group. The order of the weights should correspond to the order of levels(as.factor(group)). Defaults to  $\sqrt{p_k}$  for all  $k=1,\ldots,K$ , where  $p_k$  is the number of coefficients in the k-th group. If penalty.factor[k] =  $\emptyset$ , then the k-th group is unpenalized, and the corresponding term is always included in the model.

S3 "formula" method: named list giving the non-negative penalty weight for terms specified in the formula. Incomplete lists are allowed. Any term that is specified in formula but not in penalty.factor will be assigned the default penalty weight of  $\sqrt{p_k}$ . If penalty factor\$z = 0, then the variable z is unpenalized and always included in the model.

penalty

Character specifying which (group) penalty to use: LASSO, MCP, or SCAD.

gamma

Penalty hyperparameter that satisfies  $\gamma > 1$  for MCP and  $\gamma > 2$  for SCAD.

Ignored for LASSO penalty.

theta

Additional ("size") parameter for negative binomial responses, where the variance function is defined as  $V(\mu) = \mu + \mu^2/\theta$ 

standardized

Logical indicating whether the predictors should be groupwise standardized. If TRUE, each column of x is mean-centered and each predictor group's design matrix is scaled to have a mean-square of one before fitting the model. Regardless of whether standardization is used, the coefficients are always returned on the original data scale.

orthogonalized Logical indicating whether the predictors should be groupwise orthogonalized. If TRUE (default), each predictor group's design matrix is orthonormalized (i.e.,  $\mathbf{X}_{k}^{\top}\mathbf{X}_{k}=n\mathbf{I}_{k}$ ) before fitting the model. Regardless of whether orthogonalization is used, the coefficients are always returned on the original data scale.

intercept

Logical indicating whether an intercept term should be included in the model. Note that the intercept is always unpenalized.

thresh

Convergence threshold (tolerance). The algorithm is determined to have converged once the maximum relative change in the coefficients is below this threshold. See "Convergence" section.

maxit

Maximum number of iterations to allow.

Additional arguments used by the default or formula method.

#### **Details**

Consider a generalized linear model of the form

$$g(\mu) = \mathbf{X}^{\top} \boldsymbol{\beta}$$

where  $\mu = E(Y|\mathbf{X})$  is the conditional expectation of the response Y given the predictor vector  $\mathbf{X}$ , the function  $g(\cdot)$  is a user-specified (invertible) link function, and  $\boldsymbol{\beta}$  are the unknown regression coefficients. Furthermore, suppose that the predictors are grouped, such as

$$\mathbf{X}^{ op}oldsymbol{eta} = \sum_{k=1}^K \mathbf{X}_k^{ op}oldsymbol{eta}_k$$

where  $\mathbf{X} = (\mathbf{X}_1, \dots, \mathbf{X}_K)$  is the grouped predictor vector, and  $\boldsymbol{\beta} = (\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_K)$  is the grouped coefficient vector.

Given n observations, this function finds the  $\beta$  that minimizes

$$L(\boldsymbol{\beta}|\mathbf{D}) + \lambda P_{\alpha}(\boldsymbol{\beta})$$

where  $L(\beta|\mathbf{D})$  is the loss function with  $\mathbf{D} = \{\mathbf{y}, \mathbf{X}\}$  denoting the observed data,  $P_{\alpha}(\beta)$  is the group elastic net penalty, and  $\lambda \geq 0$  is the regularization parameter.

The loss function has the form

$$L(\boldsymbol{\beta}|\mathbf{D}) = \frac{1}{n} \sum_{i=1}^{n} w_i \ell_i(\boldsymbol{\beta}|\mathbf{D}_i)$$

where  $w_i > 0$  are the user-supplied weights, and  $\ell_i(\beta|\mathbf{D}_i)$  is the *i*-th observation's contribution to the loss function. Note that  $\ell(\cdot) = -\log(f_Y(\cdot))$  denotes the negative log-likelihood function for the given family.

The group elastic net penalty function has the form

$$P_{\alpha}(\boldsymbol{\beta}) = \alpha P_1(\boldsymbol{\beta}) + (1 - \alpha) P_2(\boldsymbol{\beta})$$

where  $\alpha \in [0, 1]$  is the user-specified alpha value,

$$P_1(\boldsymbol{\beta}) = \sum_{k=1}^K \omega_k \|\boldsymbol{\beta}_k\|$$

is the group lasso penalty with  $\omega_k \geq 0$  denoting the k-th group's penalty.factor, and

$$P_2(\boldsymbol{\beta}) = \frac{1}{2} \sum_{k=1}^{K} \omega_k \|\boldsymbol{\beta}_k\|^2$$

is the group ridge penalty. Note that  $\|\boldsymbol{\beta}_k\|^2 = \boldsymbol{\beta}_k^\top \boldsymbol{\beta}_k$  denotes the squared Euclidean norm. When penalty %in% c("MCP", "SCAD"), the group L1 penalty  $P_1(\boldsymbol{\beta})$  is replaced by the group MCP or group SCAD penalty.

#### Value

An object of class "grpnet" with the following elements:

call matched call

a0 intercept sequence of length nlambda

beta coefficient matrix of dimension nvars by nlambda

alpha balance between the group L1 (lasso) and group L2 (ridge) penalty

lambda sequence of regularization parameter values family exponential family defining the loss function

dev.ratio proportion of (null) deviance explained for each lambda (= 1 - dev / nulldev)

nulldev null deviance for each lambda

df effective degrees of freedom for each lambda nzgrp number of non-zero groups for each lambda

nzcoef number of non-zero coefficients for each lambda

xsd standard deviation of x for each group

ylev levels of response variable (only for binomial and multinomial families)

nobs number of observations

 $\begin{array}{ll} {\rm group} & {\rm group} \ {\rm label} \ {\rm vector} \\ {\rm ngroups} & {\rm number} \ {\rm of} \ {\rm groups} \ K \end{array}$ 

npasses number of iterations for each lambda

offset logical indicating if an offset was included

args list of input argument values

formula input formula (possibly after expansion)
term.labels terms that appear in formula (if applicable)

rk.args arguments for rk.model.matrix function (if applicable)

#### S3 "formula" method

**Important**: When using the S3 "formula" method, the S3 "predict" method forms the model matrix for the predictions by applying the model formula to the new data. As a result, to ensure that the corresponding S3 "predict" method works correctly, some formulaic features should be avoided.

**Polynomials:** When including polynomial terms, the poly function should be used with option raw = TRUE. Default use of the poly function (with raw = FALSE) will work for fitting the model, but will result in invalid predictions for new data. Polynomials can also be included via the I function, but this isn't recommended because the polynomials terms wouldn't be grouped together, i.e., the terms x and  $I(x^2)$  would be treated as two separate groups of size one instead of a single group of size two.

**Splines**: B-splines (and other spline bases) can be included via the S3 "formula" method. However, to ensure reasonable predictions for new data, it is necessary to specify the knots directly. For example, if x is a vector with entries between zero and one, the code bs(x, df = 5) will \*not\* produce valid predictions for new data, but the code bs(x, knots = c(0.25, 0.5, 0.75), Boundary.knots = c(0, 1)) will work as intended. Instead of attempting to integrate a call to bs() or rk() into the model formula, it is recommended that splines be included via the use.rk = TRUE argument.

## Family argument and link functions

Unlike the glm function, the family argument of the grpnet function

- \* should be a character vector (not a family object)
- \* does not allow for specification of a link function

Currently, there is only one available link function for each family:

- \* gaussian (identity):  $\mu = \mathbf{X}^{\top} \boldsymbol{\beta}$
- \* binomial (logit):  $\log(\frac{\pi}{1-\pi}) = \mathbf{X}^{\top} \boldsymbol{\beta}$
- \* multinomial (symmetric):  $\pi_{\ell} = \frac{\exp(\mathbf{X}^{\top} \boldsymbol{\beta}_{\ell})}{\sum_{l=1}^{m} \exp(\mathbf{X}^{\top} \boldsymbol{\beta}_{l})}$
- \* poisson (log):  $\log(\mu) = \mathbf{X}^{\top} \boldsymbol{\beta}$
- \* negative.binomial (log):  $\log(\mu) = \mathbf{X}^{\top} \boldsymbol{\beta}$
- \* Gamma (log):  $\log(\mu) = \mathbf{X}^{\top} \boldsymbol{\beta}$
- \* inverse.gaussian (log):  $\log(\mu) = \mathbf{X}^{\top} \boldsymbol{\beta}$

#### Binomial and multinomial

For "binomial" responses, three different possibilities exist for the input response:

- 1. vector coercible into a factor with two levels
- 2. matrix with two columns (# successes, # failures)
- 3. numeric vector with entries between 0 and 1

In this case, the weights argument should be used specify the total number of trials.

For "multinomial" responses, two different possibilities exist for the input reponse:

- 1. vector coercible into a factor with more than two levels
- 2. matrix of integers (counts) for each category level

## Convergence

The algorithm is determined to have converged once

$$\max_{j} \frac{|\beta_{j} - \beta_{j}^{\text{old}}|}{1 + |\beta_{j}^{\text{old}}|} < \epsilon$$

where  $j \in \{1, \dots, p\}$  and  $\epsilon$  is the thresh argument.

## Note

The syntax of (the default S3 method for) this function closely mimics that of the glmnet function in the **glmnet** package (Friedman, Hastie, & Tibshirani, 2010).

## Author(s)

Nathaniel E. Helwig <a href="mailto:helwig@umn.edu">helwig@umn.edu</a>

## References

Breheny, P., & Huang, J. (2015). Group descent algorithms for nonconvex penalized linear and logistic regression models with grouped predictors. *Statistics and Computing*, 25, 173-187. doi:10.1007/s1122201394242

Friedman, J., Hastie, T., & Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. *Journal of Statistical Software*, 33(1), 1-22. doi:10.18637/jss.v033.i01

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. *Journal of Computational and Graphical Statistics*. doi:10.1080/10618600.2024.2362232

Yang, Y., & Zou, H. (2015). A fast unified algorithm for solving group-lasso penalize learning problems. *Statistics and Computing*, 25, 1129-1141. doi:10.1007/s1122201494985

#### See Also

```
plot.grpnet for plotting the regularization path
predict.grpnet for predicting from grpnet objects
cv.grpnet for k-fold cross-validation of lambda
```

## **Examples**

```
######**
                  family = "gaussian"
                                         #####***######
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto)</pre>
# print regularization path info
mod
# plot coefficient paths
plot(mod)
                 family = "binomial"
#####***#####
                                         ######***######
# load data
data(auto)
# define response (1 = American, 0 = other)
y <- ifelse(auto$origin == "American", 1, 0)
# define predictors
x <- rk.model.matrix(~ 0 + ., data = auto[,1:7])
# define group
g <- attr(x, "assign")</pre>
# fit model (default method, response = y)
mod <- grpnet(x, y, g, family = "binomial")</pre>
# print regularization path info
mod
```

```
# plot coefficient paths
plot(mod)
#####***#####
                 family = "poisson"
                                      ######**
# load data
data(auto)
# fit model (formula method, response = horsepower)
mod <- grpnet(horsepower ~ ., data = auto, family = "poisson")</pre>
# print regularization path info
mod
# plot coefficient paths
plot(mod)
                 family = "negative.binomial" #####***#####
#####***#####
# load data
data(auto)
# fit model (formula method, response = horsepower)
mod \leftarrow grpnet(horsepower \sim ., data = auto, family = "negative.binomial", theta = 100)
# print regularization path info
mod
# plot coefficient paths
plot(mod)
######**
                family = "multinomial" #####***#####
# load data
data(auto)
# fit model (formula method, response = origin)
mod <- grpnet(origin ~ ., data = auto, family = "multinomial")</pre>
# print regularization path info
# plot coefficient paths
plot(mod)
```

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```
######**
                 family = "Gamma"
                                     #####***######
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto, family = "Gamma")</pre>
# print regularization path info
mod
# plot coefficient paths
plot(mod)
                 family = "inverse.gaussian"
######**
                                                #####***######
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto, family = "inverse.gaussian")</pre>
# print regularization path info
# plot coefficient paths
plot(mod)
```

plot.cv.grpnet

Plot Cross-Validation Curve for cv.grpnet Fits

## **Description**

Plots the mean cross-validation error, along with lower and upper standard deviation curves, as a function of log(lambda).

## Usage

```
## S3 method for class 'cv.grpnet'
plot(x, sign.lambda = 1, nzero = TRUE, ...)
```

## **Arguments**

x Object of class "cv.grpnet"

sign.lambda Default plots log(lambda) on the x-axis. Set to -1 to plot -1\*log(lambda) on

the x-axis instead.

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nzero Should the number of non-zero groups be printed on the top of the x-axis?

Additional arguments passed to the plot function.

#### **Details**

Produces cross-validation plot only (i.e., nothing is returned).

## Value

No return value (produces a plot)

#### Note

Syntax and functionality were modeled after the plot.cv.glmnet function in the **glmnet** package (Friedman, Hastie, & Tibshirani, 2010).

## Author(s)

Nathaniel E. Helwig <a href="mailto:helwig@umn.edu">helwig@umn.edu</a>

## References

Friedman, J., Hastie, T., & Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. *Journal of Statistical Software*, *33*(1), 1-22. doi:10.18637/jss.v033.i01

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. *Journal of Computational and Graphical Statistics*. doi:10.1080/10618600.2024.2362232

## See Also

```
cv.grpnet for k-fold cross-validation of lambda
plot.grpnet for plotting the regularization path
```

## **Examples**

```
# see 'cv.grpnet' for plotting examples
?cv.grpnet
```

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--	-------------	--

# Description

Creates a profile plot of the reguarlization paths for a fit group elastic net regularized GLM (grpnet) object.

## Usage

```
## S3 method for class 'grpnet'
plot(x, type = c("coef", "imp", "norm", "znorm"),
    newx, newdata, intercept = FALSE,
    color.by.group = TRUE, col = NULL, ...)
```

## **Arguments**

x	Object of class "grpnet"
type	What to plot on the Y-axis: "coef" for coefficient values, "imp" for importance of each group's contribution, "norm" for L2 norm of coefficients for each group, or "znorm" for L2 norm of standardized coefficients for each group.
newx	Matrix of new x scores for prediction (default S3 method). Ignored unless type = "imp".
newdata	Data frame of new data scores for prediction (S3 "formula" method). Ignored unless type = "imp".
intercept	Should the intercept be included in the plot?
color.by.group	If TRUE (default), the coefficient paths are colored according to their group membership using the colors in col. If FALSE, all coefficient paths are plotted the same color.
col	If color.by.group = TRUE, this should be a vector of length $K$ giving a color label for each group. If color.by.group = FASLE, this should be a character specifying a single (common) color. Default of col = NULL is the same as col = 1:K or col = "black".
	Additional arguments passed to the plot function.

## **Details**

Syntax and functionality were modeled after the plot.glmnet function in the **glmnet** package (Friedman, Hastie, & Tibshirani, 2010).

## Value

Produces a profile plot showing the requested type (y-axis) as a function of log(lambda) (x-axis).

## Note

If x is a multinomial model, the coefficients for each response class are plotted in a separate plot.

#### Author(s)

Nathaniel E. Helwig <helwig@umn.edu>

#### References

Friedman, J., Hastie, T., & Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. *Journal of Statistical Software*, *33*(1), 1-22. doi:10.18637/jss.v033.i01

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. *Journal of Computational and Graphical Statistics*. doi:10.1080/10618600.2024.2362232

## See Also

```
grpnet for fitting grpnet regularization paths
plot.cv.grpnet for plotting cv.grpnet objects
```

## **Examples**

```
# see 'grpnet' for plotting examples
?grpnet
```

predict.cv.grpnet

Predict Method for cv.grpnet Fits

## Description

Obtain predictions from a cross-validated group elastic net regularized GLM (cv.grpnet) object.

## Usage

#### **Arguments**

Object of class "cv.grpnet" object Matrix of new x scores for prediction (default S3 method). Must have p columns newx arranged in the same order as the x matrix used to fit the model. Data frame of new data scores for prediction (S3 "formula" method). Must newdata contain all variables in the formula used to fit the model. Lambda value(s) at which predictions should be obtained. Can input a character s ("lambda.min" or "lambda.1se") or a numeric vector. Default of "lambda.min" uses the lambda value that minimizes the mean cross-validated error. Type of prediction to return. "link" gives predictions on the link scale  $(\eta)$ . "retype sponse" gives predictions on the mean scale  $(\mu)$ . "terms" gives the predictions for each term (group) in the model  $(\eta_k)$ . "class" gives predicted class labels (for "binomial" and "multinomial" families). "coefficients" returns the coefficients used for predictions. "nonzero" returns a list giving the indices of non-zero coefficients for each s. "ncoefs" returns the number of non-zero coefficients for each s. "ngroups" returns the number of non-zero groups for each s. "norm" returns the L2 norm of each group's (raw) coefficients for each s. "znorm" returns the L2 norm of each group's standardized coefficients for each s.

#### **Details**

Predictions are calculated from the grpnet object fit to the full sample of data, which is stored as object\$grpnet.fit

See predict.grpnet for further details on the calculation of the different types of predictions.

Additional arguments (ignored)

#### Value

Depends on three factors...

- 1. the exponential family distribution
- 2. the length of the input s
- 3. the type of prediction requested

See predict.grpnet for details

#### Note

Syntax is inspired by the predict.cv.glmnet function in the **glmnet** package (Friedman, Hastie, & Tibshirani, 2010).

#### Author(s)

Nathaniel E. Helwig <a href="mailto:helwig@umn.edu">helwig@umn.edu</a>

## References

Friedman, J., Hastie, T., & Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. *Journal of Statistical Software*, *33*(1), 1-22. doi:10.18637/jss.v033.i01

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. *Journal of Computational and Graphical Statistics*. doi:10.1080/10618600.2024.2362232

#### See Also

```
cv.grpnet for k-fold cross-validation of lambda
predict.grpnet for predicting from grpnet objects
```

## **Examples**

```
######**
                 family = "gaussian"
                                        ######**
# load data
data(auto)
# 10-fold cv (formula method, response = mpg)
set.seed(1)
mod <- cv.grpnet(mpg ~ ., data = auto, alpha = 1)</pre>
# get fitted values at "lambda.min"
fit.min <- predict(mod, newdata = auto)</pre>
# get fitted values at "lambda.1se"
fit.1se <- predict(mod, newdata = auto, s = "lambda.1se")</pre>
# compare rmse for two solutions
sqrt(mean((auto$mpg - fit.min)^2))
sqrt(mean((auto$mpg - fit.1se)^2))
######**
                 family = "binomial"
                                        ######***######
# load data
data(auto)
# define response (1 = American, 0 = other)
y <- ifelse(auto$origin == "American", 1, 0)
# define predictors
x <- rk.model.matrix(~ 0 + ., data = auto[,1:7])
# define group
g <- attr(x, "assign")</pre>
# 10-fold cv (default method, response = y)
```

```
set.seed(1)
mod <- cv.grpnet(x, y, g, family = "binomial", alpha = 1)</pre>
# get fitted values at "lambda.min"
fit.min <- predict(mod, newx = x, type = "response")</pre>
# get fitted values at "lambda.1se"
fit.1se <- predict(mod, newx = x, type = "response", s = "lambda.1se")</pre>
# compare rmse for two solutions
sqrt(mean((y - fit.min)^2))
sqrt(mean((y - fit.1se)^2))
# get predicted classes at "lambda.min"
fit.min <- predict(mod, newx = x, type = "class")</pre>
# get predicted classes at "lambda.1se"
fit.1se <- predict(mod, newx = x, type = "class", s = "lambda.1se")</pre>
# compare misclassification rate for two solutions
1 - mean(y == fit.min)
1 - mean(y == fit.1se)
#####***##### family = "poisson" #####***#####
# load data
data(auto)
# 10-fold cv (formula method, response = horsepower)
set.seed(1)
mod <- cv.grpnet(horsepower ~ ., data = auto, family = "poisson", alpha = 1)</pre>
# get fitted values at "lambda.min"
fit.min <- predict(mod, newdata = auto, type = "response")</pre>
# get fitted values at "lambda.1se"
fit.1se <- predict(mod, newdata = auto, type = "response", s = "lambda.1se")</pre>
# compare rmse for two solutions
sqrt(mean((auto$horsepower - fit.min)^2))
sqrt(mean((auto$horsepower - fit.1se)^2))
######**
                 family = "negative.binomial" #####***#####
# load data
data(auto)
# 10-fold cv (formula method, response = horsepower)
set.seed(1)
```

```
mod <- cv.grpnet(horsepower ~ ., data = auto, family = "negative.binomial",</pre>
                 alpha = 1, theta = 100)
# get fitted values at "lambda.min"
fit.min <- predict(mod, newdata = auto, type = "response")</pre>
# get fitted values at "lambda.1se"
fit.1se <- predict(mod, newdata = auto, type = "response", s = "lambda.1se")</pre>
# compare rmse for two solutions
sqrt(mean((auto$horsepower - fit.min)^2))
sqrt(mean((auto$horsepower - fit.1se)^2))
#####***##### family = "multinomial" #####***#####
# load data
data(auto)
# 10-fold cv (formula method, response = origin)
set.seed(1)
mod <- cv.grpnet(origin ~ ., data = auto, family = "multinomial", alpha = 1)</pre>
# get predicted classes at "lambda.min"
fit.min <- predict(mod, newdata = auto, type = "class")</pre>
# get predicted classes at "lambda.1se"
fit.1se <- predict(mod, newdata = auto, type = "class", s = "lambda.1se")</pre>
# compare misclassification rate for two solutions
1 - mean(auto$origin == fit.min)
1 - mean(auto$origin == fit.1se)
######***##### family = "Gamma" #####***#####
# load data
data(auto)
# 10-fold cv (formula method, response = origin)
set.seed(1)
mod <- cv.grpnet(mpg ~ ., data = auto, family = "Gamma", alpha = 1)</pre>
# get fitted values at "lambda.min"
fit.min <- predict(mod, newdata = auto, type = "response")</pre>
# get fitted values at "lambda.1se"
fit.1se <- predict(mod, newdata = auto, type = "response", s = "lambda.1se")</pre>
# compare rmse for two solutions
sqrt(mean((auto$mpg - fit.min)^2))
```

```
sqrt(mean((auto$mpg - fit.1se)^2))

######****##### family = "inverse.gaussian" #####***#####

# load data
data(auto)

# 10-fold cv (formula method, response = origin)
set.seed(1)
mod <- cv.grpnet(mpg ~ ., data = auto, family = "inverse.gaussian", alpha = 1)

# get fitted values at "lambda.min"
fit.min <- predict(mod, newdata = auto, type = "response")

# get fitted values at "lambda.1se"
fit.1se <- predict(mod, newdata = auto, type = "response", s = "lambda.1se")

# compare rmse for two solutions
sqrt(mean((auto$mpg - fit.min)^2))
sqrt(mean((auto$mpg - fit.1se)^2))</pre>
```

predict.grpnet

Predict Method for grpnet Fits

# Description

Obtain predictions from a fit group elastic net regularized GLM (grpnet) object.

# Usage

## **Arguments**

object

Object of class "grpnet"

newx Matrix of new x scores for prediction (default S3 method). Must have p columns

arranged in the same order as the x matrix used to fit the model. Ignored for the

last six types of predictions.

newdata Data frame of new data scores for prediction (S3 "formula" method). Must

contain all variables in the formula used to fit the model. Ignored for the last

six types of predictions.

Lambda value(s) at which predictions should be obtained. Default uses s =

object\$lambda. Interpolation is used for s values that are not included in

object\$lambda.

type Type of prediction to return. "link" gives predictions on the link scale ( $\eta$ ). "re-

sponse" gives predictions on the mean scale  $(\mu)$ . "class" gives predicted class labels (for "binomial" and "multinomial" families). "terms" gives the predictions for each term (group) in the model  $(\eta_k)$ . "importance" gives the variable importance index for each term (group) in the model. "coefficients" returns the coefficients used for predictions. "nonzero" returns a list giving the indices of non-zero coefficients for each s. "groups" returns a list giving the labels of nonzero groups for each s. "ncoefs" returns the number of non-zero coefficients for each s. "norm" returns the L2 norm of each group's (raw) coefficients for each s. "znorm" returns

the L2 norm of each group's standardized coefficients for each s.

... Additional arguments (ignored)

#### **Details**

s

When type == "link", the predictions for each  $\lambda$  have the form

$$oldsymbol{\eta}_{\lambda} = \mathbf{X}_{\mathrm{new}} oldsymbol{eta}_{\lambda}$$

where  $\mathbf{X}_{\text{new}}$  is the argument newx (or the design matrix created from newdata by applying object\$formula) and  $\beta_{\lambda}$  is the coefficient vector corresponding to  $\lambda$ .

When type == "response", the predictions for each  $\lambda$  have the form

$$\boldsymbol{\mu}_{\lambda} = g^{-1}(\boldsymbol{\eta}_{\lambda})$$

where  $g^{-1}(\cdot)$  is the inverse link function stored in object\$family\$linkinv.

When type == "class", the predictions for each  $\lambda$  have the form

$$\mathbf{y}_{\lambda} = \arg \max_{l} \boldsymbol{\mu}_{\lambda}(l)$$

where  $\mu_{\lambda}(l)$  gives the predicted probability that each observation belongs to the l-th category (for l = 1, ..., m) using the regularization parameter  $\lambda$ .

When type == "terms", the groupwise predictions for each  $\lambda$  have the form

$$oldsymbol{\eta}_{k\lambda} = \mathbf{X}_k^{ ext{(new)}} oldsymbol{eta}_{k\lambda}$$

where  $\mathbf{X}_k^{(\mathrm{new})}$  is the portion of the argument newx (or the design matrix created from newdata by applying object\$formula) that corresponds to the k-th term/group, and  $\boldsymbol{\beta}_{k\lambda}$  are the corresponding coefficients.

When type == "importance", the variable importance indices are defined as

$$\pi_k = \left( oldsymbol{\eta}_{k\lambda}^{ op} \mathbf{C} oldsymbol{\eta}_{0\lambda} 
ight) \left( oldsymbol{\eta}_{0\lambda}^{ op} \mathbf{C} oldsymbol{\eta}_{0\lambda} 
ight)^{-1}$$

where  $\mathbf{C} = (\mathbf{I}_n - \frac{1}{n} \mathbf{1}_n \mathbf{1}_n^{\top})$  denotes a centering matrix, and  $\boldsymbol{\eta}_{0\lambda} = \sum_{k=1}^K \boldsymbol{\eta}_{k\lambda}$ . Note that  $\sum_{k=1}^K \pi_k = 1$ , but some  $\pi_k$  could be negative. When they are positive,  $\pi_k$  gives the approximate proportion of model (explained) variation that is attributed to the k-th term.

#### Value

Depends on three factors...

- 1. the exponential family distribution
- 2. the length of the input s
- 3. the type of prediction requested

For most response variables, the typical output will be...

- \* a matrix of dimension c(newnobs, length(s)) if length(s) > 1
- \* a vector of length newnobs if length(s) == 1

For multinomial response variables, the typical output will be...

- \* an array of dimension c(newnobs, length(object\$ylev), length(s)) if type
  %in% c("link", "response")
- \* a matrix of dimension c(newobs, length(s)) if type == "class"

Note: if type == "class", then the output will be the same class as object\$ylev. Otherwise, the output will be real-valued (or integer for the counts).

If type == "terms" and family != "multinomial", the output will be...

- \* an array of dimension c(newnobs, nterms, length(s)) if length(s) > 1
- \* a matrix of dimension c(newnobs, nterms) if length(s) == 1

If type == "terms" and family == "multinomial", the output will be a list of length length(object\$ylev) where each element gives the terms for the corresponding response class.

If type == "importance" and family != "multinomial", the output will be...

- \* a matrix of dimension c(nterms, length(s)) if length(s) > 1
- \* a vector of length nterms if length(s) == 1

If type == "importance" and family == "multinomial", the output will be a list of length length(object\$ylev) where each element gives the importance for the corresponding response class. If length(s) == 1, the output will be simplified to matrix.

If type == "coefficients", the output will be the same as that produced by coef.grpnet.

If type == "nonzero", the output will be a list of length length(s) where each element is a vector of integers (indices).

If type == "groups", the output will be a list of length length(s) where each element is a vector of characters (term.labels).

If type %in% c("ncoefs", "ngroups"), the output will be a vector of length length(s) where each element is an integer.

If type == "norm", the output will be a matrix of dimension c(K, length(s)), where each cell gives the L2 norm for the corresponding group and smoothing parameter. Note that K denotes the number of groups.

#### Note

Some internal code (e.g., used for the interpolation) is borrowed from the predict.glmnet function in the **glmnet** package (Friedman, Hastie, & Tibshirani, 2010).

#### Author(s)

Nathaniel E. Helwig <a href="mailto:helwig@umn.edu">helwig@umn.edu</a>

#### References

Friedman, J., Hastie, T., & Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. *Journal of Statistical Software*, *33*(1), 1-22. doi:10.18637/jss.v033.i01

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. *Journal of Computational and Graphical Statistics*. doi:10.1080/10618600.2024.2362232

#### See Also

```
grpnet for fitting grpnet regularization paths
predict.cv.grpnet for predicting from cv.grpnet objects
```

## **Examples**

```
######***######
                 family = "gaussian"
                                         ######**
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto)</pre>
# get fitted values for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newdata = auto)</pre>
# get fitted values at 3 particular points (output = 392 x 3 matrix)
fit.some <- predict(mod, newdata = auto, s = c(1.5, 1, 0.5))
# compare rmse for solutions
rmse.path <- sqrt(colMeans((auto$mpg - fit.path)^2))</pre>
rmse.some <- sqrt(colMeans((auto$mpg - fit.some)^2))</pre>
plot(log(mod\$lambda), rmse.path, cex = 0.5)
points(log(c(1.5, 1, 0.5)), rmse.some, pch = 0, col = "red")
                 family = "binomial"
#####***#####
                                         ######***######
```

```
# load data
data(auto)
# define response (1 = American, 0 = other)
y <- ifelse(auto$origin == "American", 1, 0)
# define predictors
x \leftarrow model.matrix(~., data = auto[,1:7])[,-1]
# define group (according to colnames(x))
g \leftarrow c(1, 2, 2, 2, 2, 3, 4, 5, 6, 7)
# fit model (default method, response = y)
mod <- grpnet(x, y, g, family = "binomial")</pre>
# get predicted classes for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newx = x, type = "class")</pre>
# get predicted classes at 3 particular points (output = 392 x 3 matrix)
fit.some <- predict(mod, newx = x, type = "class", s = c(.15, .1, .05))
# compare misclassification rate for solutions
miss.path <- 1 - colMeans(y == fit.path)
miss.some <- 1 - colMeans(y == fit.some)</pre>
plot(log(mod\$lambda), miss.path, cex = 0.5)
points(log(c(.15, .1, .05)), miss.some, pch = 0, col = "red")
######**
                 family = "poisson"
                                        ######**
# load data
data(auto)
# fit model (formula method, response = horsepower)
mod <- grpnet(horsepower ~ ., data = auto, family = "poisson")</pre>
# get fitted values for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newdata = auto, type = "response")</pre>
# get fitted values at 3 particular points (output = 392 x 3 matrix)
fit.some <- predict(mod, newdata = auto, type = "response", s = c(15, 10, 5))
# compare rmse for solutions
rmse.path <- sqrt(colMeans((auto$horsepower - fit.path)^2))</pre>
rmse.some <- sqrt(colMeans((auto$horsepower - fit.some)^2))</pre>
plot(log(mod\$lambda), rmse.path, cex = 0.5)
points(log(c(15, 10, 5)), rmse.some, pch = 0, col = "red")
#####***##### family = "negative.binomial" #####***#####
```

```
# load data
data(auto)
# fit model (formula method, response = horsepower)
mod <- grpnet(horsepower ~ ., data = auto, family = "negative.binomial", theta = 100)
# get fitted values for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newdata = auto, type = "response")</pre>
# get fitted values at 3 particular points (output = 392 x 3 matrix)
fit.some <- predict(mod, newdata = auto, type = "response", s = c(15, 10, 5))
# compare rmse for solutions
rmse.path <- sqrt(colMeans((auto$horsepower - fit.path)^2))</pre>
rmse.some <- sqrt(colMeans((auto$horsepower - fit.some)^2))</pre>
plot(log(mod\$lambda), rmse.path, cex = 0.5)
points(log(c(15, 10, 5)), rmse.some, pch = 0, col = "red")
                 family = "multinomial"
######**
                                            ######**
# load data
data(auto)
# fit model (formula method, response = origin)
mod <- grpnet(origin ~ ., data = auto, family = "multinomial")</pre>
# get predicted classes for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newdata = auto, type = "class")</pre>
# get predicted classes at 3 particular points (output = 392 x 3 matrix)
fit.some <- predict(mod, newdata = auto, type = "class", s = c(.1, .01, .001))
# compare misclassification rate for solutions
miss.path <- 1 - colMeans(auto$origin == fit.path)</pre>
miss.some <- 1 - colMeans(auto$origin == fit.some)</pre>
plot(log(mod\$lambda), miss.path, cex = 0.5)
points(log(c(.1, .01, .001)), miss.some, pch = 0, col = "red")
                 family = "Gamma"
######**
                                     ######**
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto, family = "Gamma")</pre>
# get fitted values for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newdata = auto, type = "response")</pre>
```

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```
# get fitted values at 3 particular points (output = 392 x 3 matrix)
fit.some <- predict(mod, newdata = auto, type = "response", s = c(0.2, 0.1, 0.05))
# compare rmse for solutions
rmse.path <- sqrt(colMeans((auto$mpg - fit.path)^2))</pre>
rmse.some <- sqrt(colMeans((auto$mpg - fit.some)^2))</pre>
plot(log(mod\$lambda), rmse.path, cex = 0.5)
points(log(c(0.2, 0.1, 0.05)), rmse.some, pch = 0, col = "red")
                 family = "inverse.gaussian"
######**
                                                 ######**
# load data
data(auto)
# fit model (formula method, response = mpg)
mod <- grpnet(mpg ~ ., data = auto, family = "inverse.gaussian")</pre>
# get fitted values for regularization path (output = 392 x 100 matrix)
fit.path <- predict(mod, newdata = auto, type = "response")</pre>
# get fitted values at 3 particular points (output = 392 x 3 matrix)
fit.some <- predict(mod, newdata = auto, type = "response", s = c(0.2, 0.1, 0.05))
# compare rmse for solutions
rmse.path <- sqrt(colMeans((auto$mpg - fit.path)^2))</pre>
rmse.some <- sqrt(colMeans((auto$mpg - fit.some)^2))</pre>
plot(log(mod\$lambda), rmse.path, cex = 0.5)
points(log(c(0.2, 0.1, 0.05)), rmse.some, pch = 0, col = "red")
```

print

S3 'print' Methods for grpnet

## Description

Prints some basic information about the coefficients (for coef.grpnet objects), observed cross-validation error (for cv.grpnet objects), or the computed regularization path (for grpnet objects).

## Usage

```
## S3 method for class 'coef.grpnet'
print(x, ...)
## S3 method for class 'cv.grpnet'
print(x, digits = max(3, getOption("digits") - 3), ...)
## S3 method for class 'grpnet'
print(x, ...)
```

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# Arguments

X	an object of class coef.grpnet, cv.grpnet, or grpnet
digits	the number of digits to print (must be a positive integer)
	additional arguments for print (currently ignored)

#### **Details**

For coef.grpnet objects, prints the non-zero coefficients and uses "." for coefficients shrunk to zero

For cv. grpnet objects, prints the function call, the cross-validation type.measure, and a two-row table with information about the min and 1se solutions.

For grpnet objects, prints a data frame with columns

- \* nGrp: number of non-zero groups for each lambda
- \* Df: effective degrees of freedom for each lambda
- \* %Dev: percentage of null deviance explained for each lambda
- \* Lambda: the values of lambda

#### Value

No return value (produces a printout)

# Note

Some syntax and functionality were modeled after the print functions in the **glmnet** package (Friedman, Hastie, & Tibshirani, 2010).

# Author(s)

Nathaniel E. Helwig <helwig@umn.edu>

## References

Friedman, J., Hastie, T., & Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. *Journal of Statistical Software*, *33*(1), 1-22. doi:10.18637/jss.v033.i01

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. *Journal of Computational and Graphical Statistics*. doi:10.1080/10618600.2024.2362232

## See Also

```
coef.grpnet for extracting coefficients
cv.grpnet for k-fold cross-validation of lambda
grpnet for fitting grpnet regularization paths
```

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## **Examples**

```
# see 'coef.grpnet' for coefficient printing examples
?coef.grpnet

# see 'cv.grpnet' for cross-validation error printing examples
?cv.grpnet

# see 'grpnet' for regularization path printing examples
?grpnet
```

rk

Reproducing Kernel Basis

# Description

Generate a reproducing kernel basis matrix for a nominal, ordinal, or polynomial smoothing spline.

# Usage

```
rk(x, df = NULL, knots = NULL, m = NULL, intercept = FALSE,
Boundary.knots = NULL, warn.outside = TRUE,
periodic = FALSE, xlev = levels(x))
```

## **Arguments**

x	the predictor vector of length n. Can be a factor, integer, or numeric, see Note.
df	the degrees of freedom, i.e., number of knots to place at quantiles of $x$ . Defaults to 5 but ignored if knots are provided.
knots	the breakpoints (knots) defining the spline. If knots are provided, the df is defined as length(unique(c(knots, Boundary.knots))).
m	the derivative penalty order: $0 = \text{ordinal spline}$ , $1 = \text{linear spline}$ , $2 = \text{cubic spline}$ , $3 = \text{quintic spline}$
intercept	should an intercept be included in the basis?
${\tt Boundary.knots}$	the boundary points for spline basis. Defaults to range(x).
warn.outside	if TRUE, a warning is provided when $\boldsymbol{x}$ values are outside of the Boundary . knots
periodic	should the spline basis functions be constrained to be periodic with respect to the Boundary. knots?
xlev	levels of x (only applicable if x is a factor)

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#### **Details**

Given a vector of function realizations f, suppose that  $f = X\beta$ , where X is the (unregularized) spline basis and  $\beta$  is the coefficient vector. Let Q denote the postive semi-definite penalty matrix, such that  $\beta^{\top}Q\beta$  defines the roughness penalty for the spline. See Helwig (2017) for the form of X and Q for the various types of splines.

Consider the spectral parameterization of the form  $f = Z\alpha$  where

$$Z = XQ^{-1/2}$$

is the regularized spline basis (that is returned by this function), and  $\alpha = Q^{1/2}\beta$  are the reparameterized coefficients. Note that  $X\beta = Z\alpha$  and  $\beta^{\top}Q\beta = \alpha^{\top}\alpha$ , so the spectral parameterization absorbs the penalty into the coefficients (see Helwig, 2021, 2024).

Syntax of this function is designed to mimic the syntax of the bs function.

#### Value

Returns a basis function matrix of dimension n by df (plus 1 if an intercept is included) with the following attributes:

df degrees of freedom
knots knots for spline basis
m derivative penalty order
intercept was an intercept included?
Boundary.knots boundary points of x
periodic is the basis periodic?
xlev factor levels (if applicable)

## Note

The (default) type of spline basis depends on the class of the input x object:

- \* If x is an unordered factor, then a nominal spline basis is used
- \* If x is an ordered factor (and m = NULL), then an ordinal spline basis is used
- \* If x is an integer or numeric (and m = NULL), then a cubic spline basis is used

Note that you can override the default behavior by specifying the m argument.

## Author(s)

Nathaniel E. Helwig <helwig@umn.edu>

#### References

Helwig, N. E. (2017). Regression with ordered predictors via ordinal smoothing splines. Frontiers in Applied Mathematics and Statistics, 3(15), 1-13. doi:10.3389/fams.2017.00015

Helwig, N. E. (2021). Spectrally sparse nonparametric regression via elastic net regularized smoothers. Journal of Computational and Graphical Statistics, 30(1), 182-191. doi:10.1080/10618600.2020.1806855

Helwig, N. E. (2024). Precise tensor product smoothing via spectral splines. Stats, 7(1), 34-53. doi:10.3390/stats7010003

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```
######***##### NOMINAL SPLINE BASIS #####***#####
x <- as.factor(LETTERS[1:5])</pre>
basis \leftarrow rk(x)
plot(1:5, basis[,1], t = "l", ylim = extendrange(basis))
for(j in 2:ncol(basis)){
 lines(1:5, basis[,j], col = j)
######***##### ORDINAL SPLINE BASIS #####***#####
x <- as.ordered(LETTERS[1:5])</pre>
basis \leftarrow rk(x)
plot(1:5, basis[,1], t = "l", ylim = extendrange(basis))
for(j in 2:ncol(basis)){
 lines(1:5, basis[,j], col = j)
######***##### LINEAR SPLINE BASIS #####***#####
x < - seq(0, 1, length.out = 101)
basis \leftarrow rk(x, m = 1)
plot(x, basis[,1], t = "1", ylim = extendrange(basis))
for(j in 2:ncol(basis)){
 lines(x, basis[,j], col = j)
######***##### CUBIC SPLINE BASIS ######***#####
x <- seq(0, 1, length.out = 101)
basis \leftarrow rk(x)
basis <- scale(basis) # for visualization only!</pre>
plot(x, basis[,1], t = "l", ylim = extendrange(basis))
for(j in 2:ncol(basis)){
 lines(x, basis[,j], col = j)
}
######***##### QUINTIC SPLINE BASIS #####***#####
x <- seq(0, 1, length.out = 101)
basis <- \text{rk}(x, m = 3)
basis <- scale(basis) # for visualization only!</pre>
plot(x, basis[,1], t = "l", ylim = extendrange(basis))
for(j in 2:ncol(basis)){
 lines(x, basis[,j], col = j)
```

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rk.model.matrix Construct Design Matrices via Reproducing Kernels	rk.model.matrix	Construct Design Matrices via Reproducing Kernels	
---	-----------------	---	--

## **Description**

Creates a design (or model) matrix using the rk function to expand variables via a reproducing kernel basis.

#### **Usage**

```
rk.model.matrix(object, data = environment(object), ...)
```

## **Arguments**

object a formula or terms object describing the fit model
data a data frame containing the variables referenced in object
additional arguments passed to the rk function, e.g., df, knots, m, etc. Arguments must be passed as a named list, see Examples.

## **Details**

Designed to be a more flexible alternative to the model.matrix function. The rk function is used to construct a marginal basis for each variable that appears in the input object. Tensor product interactions are formed by taking a row.kronecker product of marginal basis matrices. Interactions of any order are supported using standard formulaic conventions, see Note.

# Value

The design matrix corresponding to the input formula and data, which has the following attributes:

assign	an integer vector with an entry for each column in the matrix giving the term in the formula which gave rise to the column
term.labels	a character vector containing the labels for each of the terms in the model
knots	a named list giving the knots used for each variable in the formula
m	a named list giving the penalty order used for each variable in the formula
periodic	a named list giving the periodicity used for each variable in the formula
xlev	a named list giving the factor levels used for each variable in the formula

## Note

For formulas of the form  $y \sim x + z$ , the constructed model matrix has the form cbind(rk(x), rk(z)), which simply concatenates the two marginal basis matrices. For formulas of the form  $y \sim x$ : z, the constructed model matrix has the form row.kronecker(rk(x), rk(z)), where row.kronecker denotes the row-wise kronecker product. The formula  $y \sim x * z$  is a shorthand for  $y \sim x + z + x : z$ , which concatenates the two previous results. Unless it is suppressed (using 0+), the first column of the basis will be a column of ones named (Intercept).

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## Author(s)

Nathaniel E. Helwig <a href="mailto:helwig@umn.edu">helwig@umn.edu</a>

#### References

Helwig, N. E. (2017). Regression with ordered predictors via ordinal smoothing splines. Frontiers in Applied Mathematics and Statistics, 3(15), 1-13. doi:10.3389/fams.2017.00015

Helwig, N. E. (2021). Spectrally sparse nonparametric regression via elastic net regularized smoothers. Journal of Computational and Graphical Statistics, 30(1), 182-191. doi:10.1080/10618600.2020.1806855

Helwig, N. E. (2024). Precise tensor product smoothing via spectral splines. Stats, 7(1), 34-53. doi:10.3390/stats7010003

## See Also

See rk for details on the reproducing kernel basis

```
# load auto data
data(auto)
# additive effects
x <- rk.model.matrix(mpg ~., data = auto)
dim(x)
                            # check dimensions
attr(x, "assign")
                           # check group assignments
                         # check group ass_s
# check term labels
attr(x, "term.labels")
# two-way interactions
x <- rk.model.matrix(mpg ~ . * ., data = auto)</pre>
                   # check dimensions
dim(x)
attr(x, "assign")
                          # check group assignments
attr(x, "term.labels") # check term labels
# specify df for horsepower, weight, and acceleration
# note: default df = 5 is used for displacement and model.year
df <- list(horsepower = 6, weight = 7, acceleration = 8)</pre>
x \leftarrow rk.model.matrix(mpg \sim ., data = auto, df = df)
sapply(attr(x, "knots"), length) # check df
# specify knots for model.year
# note: default knots are selected for other variables
knots <- list(model.year = c(1970, 1974, 1978, 1982))
x <- rk.model.matrix(mpg ~ ., data = auto, knots = knots)</pre>
sapply(attr(x, "knots"), length) # check df
```

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row.kronecker

Row-Wise Kronecker Product

# Description

Calculates the row-wise Kronecker product between two matrices with the same number of rows.

# Usage

```
row.kronecker(X, Y)
```

# **Arguments**

```
X matrix of dimension n \times p
Y matrix of dimension n \times q
```

#### **Details**

```
Given X of dimension c(n, p) and Y of dimension c(n, q), this function returns cbind(x[,1]*Y, x[,2]*Y, ..., x[,p]*Y) which is a matrix of dimension c(n, p*q)
```

#### Value

Matrix of dimension  $n \times pq$  where each row contains the Kronecker product between the corresponding rows of X and Y.

# Author(s)

Nathaniel E. Helwig <helwig@umn.edu>

# See Also

```
Used by the rk.model.matrix to construct basis functions for interaction terms
See kronecker for the regular kronecker product
```

```
X <- matrix(c(1, 1, 2, 2), nrow = 2, ncol = 2)
Y <- matrix(1:6, nrow = 2, ncol = 3)
row.kronecker(X, Y)</pre>
```

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StartupMessage

Startup Message for grpnet

# **Description**

Prints the startup message when grpnet is loaded. Not intended to be called by the user.

## **Details**

The 'grpnet' ascii start-up message was created using the taag software.

# References

https://patorjk.com/software/taag/

visualize.penalty

Plots grpnet Penalty Function or its Derivative

## **Description**

Makes a plot or returns a data frame containing the group elastic net penalty (or its derivative) evaluated at a sequence of input values.

# Usage

# Arguments

```
x sequence of values at which to evaluate the penalty.

penalty which penalty or penalties should be plotted?

alpha elastic net tuning parameter (between 0 and 1).

lambda overall tuning parameter (non-negative).

gamma additional hyperparameter for MCP (>1) or SCAD (>2).
```

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derivative	if FALSE (default), then the penalty is plotted; otherwise the derivative of the penalty is plotted.
plot	if TRUE (default), then the result is plotted; otherwise the result is returned as a data frame.
subtitle	if TRUE (default), then the hyperparameter values are displayed in the subtitle.
legend	if TRUE (default), then a legend is included to distinguish the different penalty types.
location	the legend's location; ignored if legend = FALSE.
	addition arguments passed to plot function, e.g., xlim, ylim, etc.

#### **Details**

The group elastic net penalty is defined as

$$P_{\alpha,\lambda}(\boldsymbol{\beta}) = Q_{\lambda_1}(\|\boldsymbol{\beta}\|) + \frac{\lambda_2}{2}\|\boldsymbol{\beta}\|^2$$

where  $Q_{\lambda}()$  denotes the L1 penalty (LASSO, MCP, or SCAD),  $\|\boldsymbol{\beta}\| = (\boldsymbol{\beta}^{\top}\boldsymbol{\beta})^{1/2}$  denotes the Euclidean norm,  $\lambda_1 = \lambda \alpha$  is the L1 tuning parameter, and  $\lambda_2 = \lambda (1-\alpha)$  is the L2 tuning parameter. Note that  $\lambda$  and  $\alpha$  denote the lambda and alpha arguments.

## Value

If plot = TRUE, then produces a plot.

If plot = FALSE, then returns a data frame.

## Author(s)

Nathaniel E. Helwig <a href="mailto:helwig@umn.edu">helwig@umn.edu</a>

## References

Fan J, & Li, R. (2001). Variable selection via nonconcave penalized likelihood and its oracle properties. *Journal of the American Statistical Association*, 96(456), 1348-1360. doi:10.1198/016214501753382273

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. *Journal of Computational and Graphical Statistics*. doi:10.1080/10618600.2024.2362232

Tibshirani, R. (1996). Regression and shrinkage via the Lasso. *Journal of the Royal Statistical Society, Series B*, 58, 267-288. doi:10.1111/j.25176161.1996.tb02080.x

Zhang CH (2010). Nearly unbiased variable selection under minimax concave penalty. *The Annals of Statistics*, 38(2), 894-942. doi:10.1214/09AOS729

## See Also

visualize. shrink for plotting shrinkage operator

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# **Examples**

```
# plot penalty functions
visualize.penalty()

# plot penalty derivatives
visualize.penalty(derivative = TRUE)
```

visualize.shrink

Plots grpnet Shrinkage Operator or its Estimator

# **Description**

Makes a plot or returns a data frame containing the group elastic net shrinkage operator (or its estimator) evaluated at a sequence of input values.

# Usage

# **Arguments**

Х	sequence of values at which to evaluate the penalty.
penalty	which penalty or penalties should be plotted?
alpha	elastic net tuning parameter (between 0 and 1).
lambda	overall tuning parameter (non-negative).
gamma	additional hyperparameter for MCP (>1) or SCAD (>2).
fitted	if FALSE (default), then the shrinkage operator is plotted; otherwise the shrunken estimator is plotted.
plot	if TRUE (default), then the result is plotted; otherwise the result is returned as a data frame.
subtitle	if TRUE (default), then the hyperparameter values are displayed in the subtitle.
legend	if TRUE (default), then a legend is included to distinguish the different penalty types.
location	the legend's location; ignored if legend = FALSE.
	addition arguments passed to plot function, e.g., xlim, ylim, etc.

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## **Details**

The updates for the group elastic net estimator have the form

$$\boldsymbol{\beta}_{\alpha,\lambda}^{(t+1)} = S_{\lambda_1,\lambda_2}(\|\mathbf{b}_{\alpha,\lambda}^{(t+1)}\|)\mathbf{b}_{\alpha,\lambda}^{(t+1)}$$

where  $S_{\lambda_1,\lambda_2}(\cdot)$  is a shrinkage and selection operator, and

$$\mathbf{b}_{\alpha,\lambda}^{(t+1)} = \boldsymbol{\beta}_{\alpha,\lambda}^{(t)} + (\delta_{(t)}\epsilon)^{-1}\mathbf{g}^{(t)}$$

is the unpenalized update with  $\mathbf{g}^{(t)}$  denoting the current gradient.

Note that  $\lambda_1 = \lambda \alpha$  is the L1 tuning parameter,  $\lambda_2 = \lambda(1 - \alpha)$  is the L2 tuning parameter,  $\delta_{(t)}$  is an upper-bound on the weights appearing in the Fisher information matrix, and  $\epsilon$  is the largest eigenvalue of the Gramm matrix  $n^{-1}\mathbf{X}^{\top}\mathbf{X}$ .

#### Value

If plot = TRUE, then produces a plot.

If plot = FALSE, then returns a data frame.

## Author(s)

Nathaniel E. Helwig <helwig@umn.edu>

#### References

Fan J, & Li, R. (2001). Variable selection via nonconcave penalized likelihood and its oracle properties. *Journal of the American Statistical Association*, 96(456), 1348-1360. doi:10.1198/016214501753382273

Helwig, N. E. (2024). Versatile descent algorithms for group regularization and variable selection in generalized linear models. *Journal of Computational and Graphical Statistics*. doi:10.1080/10618600.2024.2362232

Tibshirani, R. (1996). Regression and shrinkage via the Lasso. *Journal of the Royal Statistical Society, Series B*, 58, 267-288. doi:10.1111/j.25176161.1996.tb02080.x

Zhang CH (2010). Nearly unbiased variable selection under minimax concave penalty. *The Annals of Statistics*, 38(2), 894-942. doi:10.1214/09AOS729

## See Also

visualize.penalty for plotting penalty function

```
# plot shrinkage operator
visualize.shrink()
```

```
# plot shrunken estimator
visualize.shrink(fitted = TRUE)
```

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