Package ‘expint’

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Type Package

Title Exponential Integral and Incomplete Gamma Function

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Description The exponential integrals E_1(x), E_2(x), E_n(x) and Ei(x), and the incomplete gamma function G(a, x) defined for negative values of its first argument. The package also gives easy access to the underlying C routines through an API; see the package vignette for details. A test package included in sub-directory example_API provides an implementation. C routines derived from the GNU Scientific Library <https://www.gnu.org/software/gsl/>.

Depends R (>= 3.3.0)

License GPL (>= 2)

URL https://gitlab.com/vigou3/expint

BugReports https://gitlab.com/vigou3/expint/-/issues

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Author Vincent Goulet [cre, aut],
Gerard Jungman [aut] (Original GSL code),
Brian Gough [aut] (Original GSL code),
Jeffrey A. Ryan [aut] (Package API),
Robert Gentleman [aut] (Parts of the R to C interface),
Ross Ihaka [aut] (Parts of the R to C interface),
R Core Team [aut] (Parts of the R to C interface),
R Foundation [aut] (Parts of the R to C interface)

Maintainer Vincent Goulet <vincent.goulet@act.ulaval.ca>

Repository CRAN

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`expint-package` *Exponential Integral and Incomplete Gamma Function*

Description

The exponential integrals $E_1(x)$, $E_2(x)$, $E_n(x)$ and $Ei(x)$, and the incomplete gamma function $\Gamma(a, x)$ that is defined for negative values of its first argument.

Details

The exponential integral

$$E_1(x) = \int_x^\infty \frac{e^{-t}}{t}\, dt$$

and the incomplete gamma function

$$\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t}\, dt$$

are closely related functions that arise in various fields of mathematics.

expint is a small package that provides R functions to compute the exponential integral and the incomplete gamma function.

Most conveniently for R package developers, the package also gives access to the underlying C workhorses through an API; see the package vignette for instructions.

The C routines are adapted versions of those of the GNU Scientific Library `https://www.gnu.org/software/gsl/`.

Author(s)

Vincent Goulet `<vincent.goulet@act.ulaval.ca>`

References

Abramowitz, M. and Stegun, I. A. (1972), *Handbook of Mathematical Functions*, Dover.

See Also

`expint` for the exponential integral family of functions.

`gammainc` for the incomplete gamma function.

`vignette("expint")` for a detailed presentation of the package.

expint — *Exponential Integral*

Description

The exponential integrals $E_1(x)$, $E_2(x)$, $E_n(x)$ and Ei.

Usage

```
expint(x, order = 1L, scale = FALSE)
expint_E1(x, scale = FALSE)
expint_E2(x, scale = FALSE)
expint_En(x, order, scale = FALSE)
expint_Ei(x, scale = FALSE)
```

Arguments

`x`	vector of real numbers.
`order`	vector of non-negative integers; see Details.
`scale`	logical; when `TRUE` the result will be scaled by e^x.

Details

Abramowitz and Stegun (1972) first define the exponential integral as

$$E_1(x) = \int_x^\infty \frac{e^{-t}}{t}\,dt, \quad x \neq 0.$$

An alternative definition (to be understood in terms of the Cauchy principal value due to the singularity of the integrand at zero) is

$$\mathrm{Ei}(x) = -\int_{-x}^\infty \frac{e^{-t}}{t}\,dt = -E_1(-x).$$

The exponential integral can also generalized to `order` n as

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n}\,dt,$$

for $n = 0, 1, 2, \ldots$; x a real number (non-negative when $n > 2$).

The following relation holds:

$$E_n(x) = x^{n-1}\Gamma(1-n, x),$$

where $\Gamma(a, x)$ is the incomplete gamma function implemented in `gammainc`.

By definition, $E_0(x) = x^{-1}e^{-x}$, $x \neq 0$.

Function `expint` is vectorized in both `x` and `order`, whereas function `expint_En` expects a single value for `order` and will only use the first value if `order` is a vector.

Non-integer values of `order` will be silently coerced to integers using truncation towards zero.

Value

The value of the exponential integral.

Invalid arguments will result in return value NaN, with a warning.

Note

The C implementation is based on code from the GNU Software Library https://www.gnu.org/software/gsl/.

Author(s)

Vincent Goulet <vincent.goulet@act.ulaval.ca>

References

Abramowitz, M. and Stegun, I. A. (1972), *Handbook of Mathematical Functions*, Dover.

See Also

gammainc

Examples

```
## See section 5.3 of Abramowitz and Stegun
expint(1.275, order = 1:10)
expint(10, order = 1:10) * 1e5
expint(c(1.275, 10), order = c(1, 2))

expint_E1(1.275)                        # same as above
expint_E2(10)                           # same as above

## Figure 5.1 of Abramowitz and Stegun
curve(expint_Ei, xlim = c(0, 1.6), ylim = c(-3.9, 3.9),
      ylab = "y")
abline(h = 0)
curve(expint_E1, add = TRUE)
x <- 1.5
text(x, c(expint_Ei(x), expint_E1(x)),
     expression(Ei(x), E[1](x)),
     adj = c(0.5, -0.5))

## Figure 5.2 of Abramowitz and Stegun
plot(NA, xlim = c(-1.6, 1.6), ylim = c(0, 1),
     xlab = "x", ylab = expression(E[n](x)))
n <- c(10, 5, 3, 2, 1, 0)
for (order in n)
    curve(expint_En(x, order), add = TRUE)
x <- c(0.1, 0.15, 0.25, 0.35, 0.5, 0.7)
text(x, expint(x, n), paste("n =", n),
     adj = c(-0.2, -0.5))
```

gammainc *Incomplete Gamma Function*

Description

The incomplete gamma function $\Gamma(a, x)$.

Usage

```
gammainc(a, x)
```

Arguments

a	vector of real numbers.
x	vector of non-negative real numbers.

Details

As defined in 6.5.3 of Abramowitz and Stegun (1972), the incomplete gamma function is

$$\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} \, dt$$

for a real and $x \geq 0$.

For non-negative values of a, we have

$$\Gamma(a, x) = \Gamma(a)(1 - P(a, x)),$$

where $\Gamma(a)$ is the function implemented by R's `gamma()` and $P(a, x)$ is the cumulative distribution function of the gamma distribution (with scale equal to one) implemented by R's `pgamma()`.

Also, $\Gamma(0, x) = E_1(x)$, $x > 0$, where $E_1(x)$ is the exponential integral implemented in `expint`.

Value

The value of the incomplete gamma function.

Invalid arguments will result in return value `NaN`, with a warning.

Note

The C implementation is based on code from the GNU Software Library https://www.gnu.org/software/gsl/.

Author(s)

Vincent Goulet <vincent.goulet@act.ulaval.ca>

References

Abramowitz, M. and Stegun, I. A. (1972), *Handbook of Mathematical Functions*, Dover.

See Also

expint

Examples

```
## a > 0
x <- c(0.2, 2.5, 5, 8, 10)
a <- 1.2
gammainc(a, x)
gamma(a) * pgamma(x, a, 1, lower = FALSE) # same

## a = 0
a <- 0
gammainc(a, x)
expint(x)                                 # same

## a < 0
a <- c(-0.25, -1.2, -2)
sapply(a, gammainc, x = x)
```

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