Weighted Support Vector Machine Formulation

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The original formulation of unweighted SVM with linear kernel is as follows Valdimir and Vapnik (1995):

$$\begin{aligned}
\min_{\omega,\xi} \quad & \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{n}(\xi_i + \xi_i^*) \\
\text{s.t.} \quad & y_i - \langle \omega, x_i \rangle - \omega_0 \le \varepsilon + \xi_i, \\
& \langle \omega, x_i \rangle + \omega_0 - y_i \le \varepsilon + \xi_i^*, \\
& \xi_i, \xi_i^* \ge 0.
\end{aligned}$$

The constant $C > 0$ determines the trade-off between the flatness of f and the amount up to which deviations larger than ε are tolerated. This corresponds to dealing with a so called ε-insensitive loss function $|\xi|_\varepsilon$ described by

$$|\xi|_\varepsilon = \begin{cases} 0, & \text{if } |\xi| \le \varepsilon \\ |\xi| - \varepsilon, & o/w. \end{cases}$$

The corresponding weighted SVM with W_i as individual weights:

$$\begin{aligned}
\min_{\omega,\xi} \quad & \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{n}W_i(\xi_i + \xi_i^*) \\
\text{s.t.} \quad & y_i - \langle \omega, x_i \rangle - \omega_0 \le \varepsilon + \xi_i, \\
& \langle \omega, x_i \rangle + \omega_0 - y_i \le \varepsilon + \xi_i^*, \\
& \xi_i, \xi_i^* \ge 0.
\end{aligned}$$

Other kinds of weighted SVMs (with different kernels) have the similar formuation.
Available kernels:

kernel	formula	parameters
linear	$\mathbf{u}^\top \mathbf{v}$	(none)
polynomial	$(\gamma \mathbf{u}^\top \mathbf{v} + c_0)^d$	γ, d, c_0
radial basis fct.	$\exp\{-\gamma\|\mathbf{u} - \mathbf{v}\|^2\}$	γ
sigmoid	$\tanh\{\gamma \mathbf{u}^\top \mathbf{v} + c_0\}$	γ, c_0

References

V Valdimir and N Vapnik. The nature of statistical learning theory. 1995.